

Anomalous dispersion of fourier components of surface gravity waves in the near shore area

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CHAPTER 12

ANOMALOUS DISPERSION OF FOURIER COMPONENTS OF SURFACE GRAVITY WAVES IN THE NEAR SHORE AREA

by

Fritz Büsching¹⁾

ABSTRACT

Water level deflections $\eta(t)$ have been measured synchronously at some positions in a beach profile on the isle of SYLT / North Sea during severe storm surge conditions as well as at attenuating wave action.

A steadily increasing wave period \bar{T}_z in the upbeach direction, turning out from strip chart evaluations, is in accordance with the result of FOURIER syntheses. Near shore wave deformation is explained by ANOMALOUS dispersion of the frequency components.

1. INTRODUCTION

Because of the well known restrictions of model investigations on surf zone processes (FÜHRBÖTER 1970 and 1971) since 1971 comprehensive field investigations have been performed by members of the LEICHTWEISS-INSTITUT on the west coast of the isle of SYLT / North Sea.

Especially the energy transformation on the beach was considered by FÜHRBÖTER (1974) based on strip chart evaluations and on a linear analysis.

Later on in this respect the spectrum analysis was used by the author (BÜSCHING, 1974, 1975, 1976).

The present study refers to both partly previously presented data sets analyzed by means of the zero-up-crossing evaluation method and by spectral functions respectively. In the first part of this study the description of the wave deformation in the upbeach direction is based on strip chart evaluations, whereas the second part consists of an attempt to explain these deformation processes by spectral methods.

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WAVE PARAMETERS IN THE NEAR SHORE ZONE

a conservative treatment of the transformation of shoaling waves into breaking and finally into broken waves the wave period is assumed to be a constant, whereas wave heights, lengths and celerities change. In addition to this it is well known that real gravity waves continuously change form as they proceed into shallower water.

This, however, is not only represented by the different asymmetries previously defined (BIESEL, 1951 ; PATRICK and WIEGEL, 1956 ; ABEYMO, 1968 ; IWAGAKI, SAKAI and KAWASHIMA, 1973 ; FÜHRBÖTER, 1974) but also by the so-called decomposition of the initial wave into two or more waves (solitons) (MULTER and GALVIN, 1967 ; MADSEN and MEI, 1969 ; ZABUSKY and GALVIN, 1971 ; GALVIN, 1972).

Applying the zero-up-crossing evaluation method on strip chart records of water level deflexions which are deformed in such a way, this method turns out to be somewhat questionable, as in the present case the result is a steadily increasing mean wave period \bar{T}_z in the upbeach direction.

This can be seen from Fig. 1 containing the variation of some wave parameters in the coast perpendicular measuring profile on the isle of SYLT for synchronous measurements at fixed positions on March 18th and April 3rd, 1973 respectively.

The shown parameters (mean wave heights \bar{H}_z and mean wave periods \bar{T}_z) refer to synchronously measured water level deflexions taken by echo sounders at the offshore positions at 225 m, 570 m, 940 m and 1280 m and by pressure type wave meters at the beach positions 40, 50, 60, 70, 80, 90 and 100 m.

The details of the wave measuring instrumentation used are given by FÜHRBÖTER and BÜSCHING (1974). Because of the lack of space in the following it shall only be considered here the measurements on the beach face, where in addition to the above mentioned parameters the wave propagation velocities could be determined from the phase differences between the respective synchronously taken wave traces.

Additional interpretation of the more offshore measurements and the influence of the long shore bar (ridge) on the wave deformation will be given in

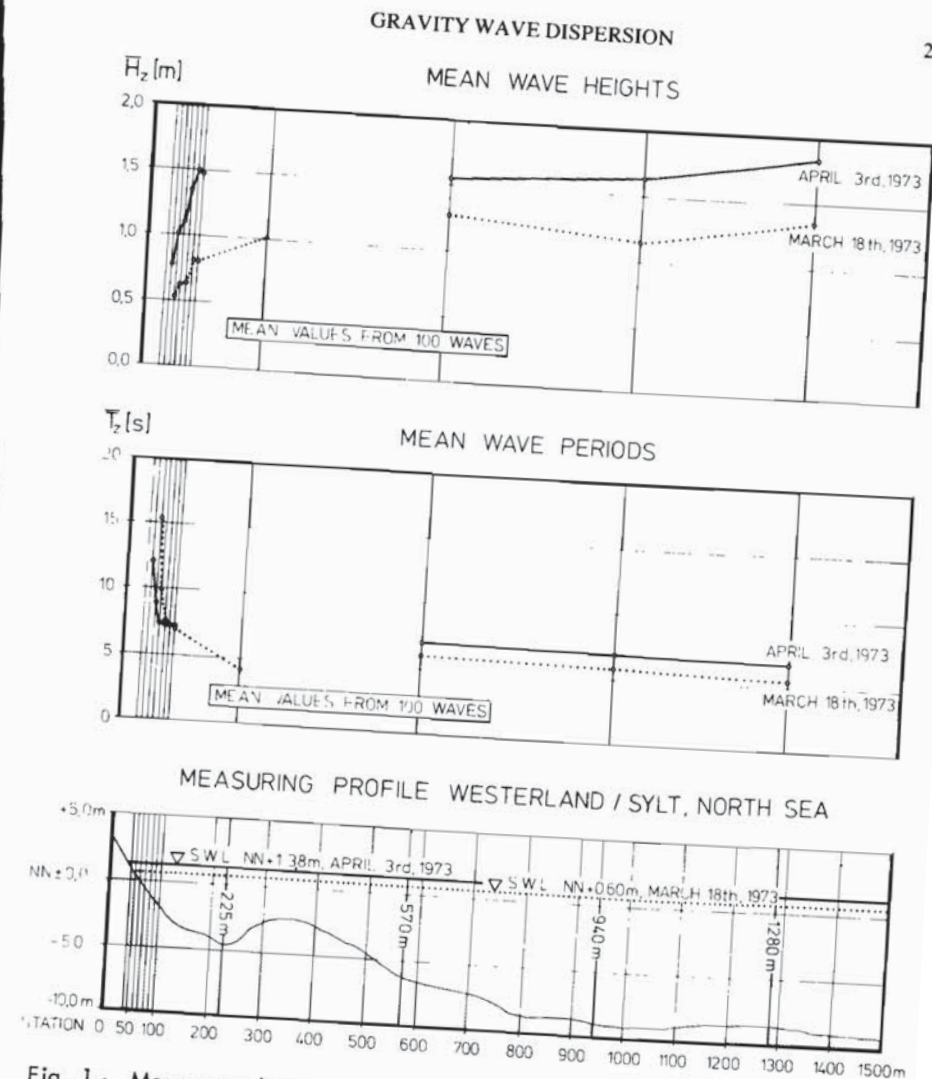


Fig. 1: Mean wave heights and periods in the measuring profile on the isle of SYLT / North Sea

another paper (BÜSCHING, 1978) with reference to the measurements of RAMAN (1976) and WANG and YANG (1976) in the same area.

As regards energy dissipation treatments based on the present measurements, they are contained in FÜHRBÖTER (1974).

The complete strip chart data are plotted in Fig. 2 with reference to the beach measuring positions. With the known wave propagation velocity the mean wave

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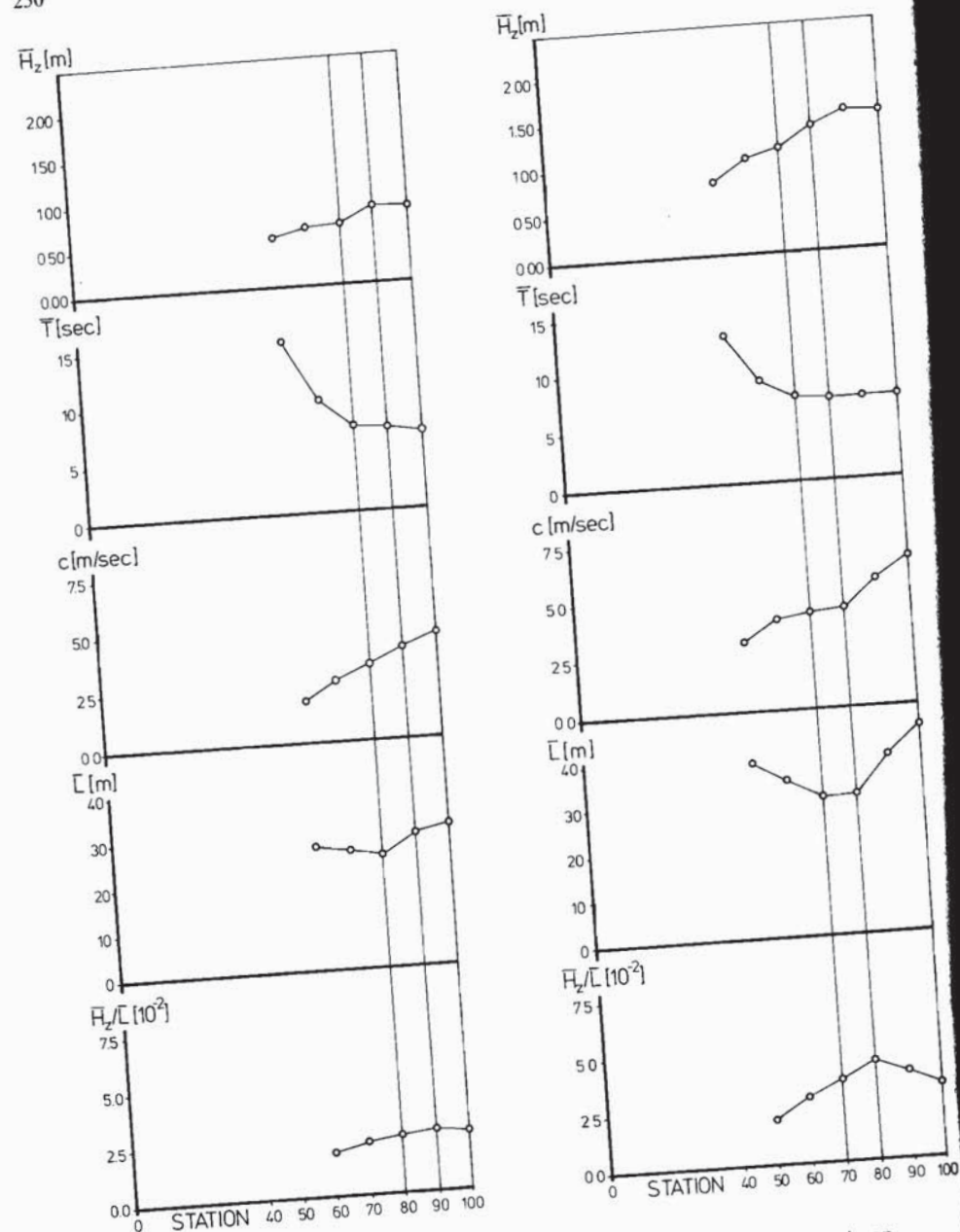


Fig. 2: Variation of wave parameters on the beach during measurements on March 18th and April 3rd, 1973 respectively

length \bar{L}_z and mean steepness \bar{H}_z/\bar{L}_z can be calculated.

As to be seen from Fig. 2 the overall impression is that mean wave lengths decrease until the maximum steepness is reached and from that moment on the wave length again increases along with decreasing wave steepness.

It is worth mentioning here that the maximum steepness values $\bar{H}_z/\bar{L}_z = 0.028$ and $\bar{H}_z/\bar{L}_z = 0.044$ respectively are both in reasonable agreement with MICHE's formula for the breaker steepness

$$H_b/L_b = 0.140 \tanh [2 \pi d_b/L_b]$$

The magnitude of underprediction by this formula is minimal and can as well be interpreted as errors in the local water depth measurements of the order of only 5 and 10 cm respectively.

In order to demonstrate the variation of wave heights (\bar{H}_z), wave lengths (\bar{L}_z) and horizontal wave asymmetries ($\alpha = \Delta L/L$ as defined by FÜHRBÖTER (1974)) in an overall view the average wave deformation is plotted for both measurements in Fig. 3.

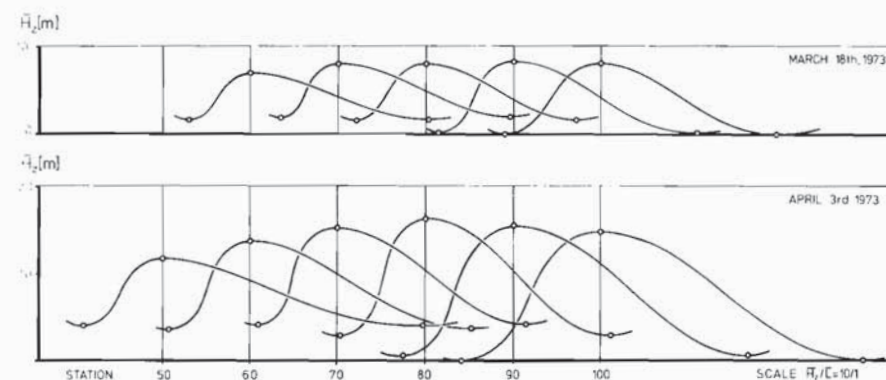


Fig. 3: Average wave deformation based on the variation of wave height \bar{H}_z , wave length \bar{L}_z and horizontal wave asymmetry α only

Constructing these sets of curves mass conservation in both cases is preserved in such a way that the initial volumes of the water mass under the waves at position 100 m were placed beneath each deformed wave contour. Thus with the wave shapes changing in the upbeach direction the water level increases.

As the construction of each wave contour is based on three points only, this procedure, however, can only deliver a crude estimate of the wave set up, which is obviously underpredicted because especially the indicated convex front faces significantly deviate from the real shapes of steep, breaking and broken waves. Further interpretation of these measurements will be given in the above mentioned additional paper.

3. SPECTRAL FUNCTIONS

As mentioned before analyses based on the zero-up-crossing or similar methods applied to strip chart records of near shore water level deflexions are valuable only to a certain extent, because deformations like the decomposition into solutions are not considered herein.

This is supposed to be the reason why the author previously was not able to establish a correlation of some quality between the significant wave period $T_z^{1/3}$ (of the zero-up-crossing-method) and the spectral peak period determined in different ways. (BÜSCHING, 1974 ; also HARRIS, 1972).

As the above demonstrated variation of wave parameters in the upbeach direction is difficult to explain in the time domain, in the following an attempt of explanation in the frequency domain is presented by means of the storm surge data of December 13th to 14th, 1973 and measurements at attenuating wave action.

This was formerly started with the investigation of synchronously measured energy spectra taken in the beach profile of Fig. 1 at positions 100 m and 85 m distant from the shoreline respectively.

The present study consists, however, in an analysis of the TRANSFER FUNCTIONS and COHERENCE FUNCTIONS based on cross power spectrum analysis. Because of the lack of space here it is not possible to go into the details of the spectrum analysis. The basical parameters, however, in this respect are equal to those mentioned in the author's previous work (BÜSCHING, 1975 and 1976).

As is well known the complex transfer function measures the relationship between any two signals at specific frequencies by relative PHASE and MAGNITUDE. In the present case the signals represent the water level deflexions $\eta_{100}(t)$ and $\eta_{85}(t)$ at positions 100 m and 85 m respectively. Hence the transfer function is defined as

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$$H_{\eta_{85} \eta_{100}}(f) = \frac{S_{\eta_{100}, \eta_{85}}}{S_{\eta_{85}, \eta_{85}}}$$

with

$$S_{\eta_{100} \eta_{85}}(f) = \text{CROSS POWER SPECTRUM from the water level deflexions at positions 100 and 85 m.}$$

and

$$S_{\eta_{85} \eta_{85}}(f) = \text{AUTO POWER SPECTRUM at position 85 m.}$$

The coherence function is constructed to detect the presence of noise (or nonlinear distortion) in the transfer functions. If the transfer system is linear (and noise-free) the input to output coherence must be 1 (BENDAT and PIER-SOL, 1968).

Because of the shape of the coherence functions in the present case the frequency range 0 to 0.36 Hz was selected for evaluation only. As an example in Fig. 4 the total set of interesting spectral functions is plotted belonging to the measuring interval at the highest storm tide water level (mean record water depth $z_3 = 3.1$ m).

In the upper part of the graph the respective energy spectra are shown, where-as the coherence function and the transfer function subdivided into magnitude and phase are plotted below.

For instance at the spectral peak at a frequency $f = 0.073$ Hz the coherence is a maximum and the magnitude of the transfer function is greater than 1. This means in the present case that - as expected - at this frequency the amplitude of the respective FOURIER COMPONENT at position 100 m is greater than that at position 85 m and on the other hand the phase difference is about 80 degrees, which can be seen from the $\varnothing(f)$ -curve plotted below. As the distance of the measuring devices $x[m]$ is known, the phase information $\varnothing(f)$ [degrees] of the transfer function can easily be converted into a PHASE VELOCITY plot $c(f)$ [m/sec]:

$$\left. \begin{aligned} \frac{\varnothing}{360^\circ} &= \frac{x}{L} \\ c = \frac{L}{T} &= L \cdot f \end{aligned} \right\} c = \frac{x \cdot 360^\circ}{\varnothing} \cdot f \text{ [m / sec]}$$

$$\begin{aligned} f &= \frac{1}{T} = \text{frequency of FOURIER component} \\ L &= \text{length of the component wave} \\ T &= \text{period of the component wave} \end{aligned}$$

This curve is also shown in the lower part of Fig. 4.

In this connexion it has to be mentioned here that the transformation of the phase information $\varnothing(f)$ - presented in the range $-180^\circ \leq \varnothing \leq +180^\circ$ only - is based on the assumption that the magnitude of the phase velocity of dominant

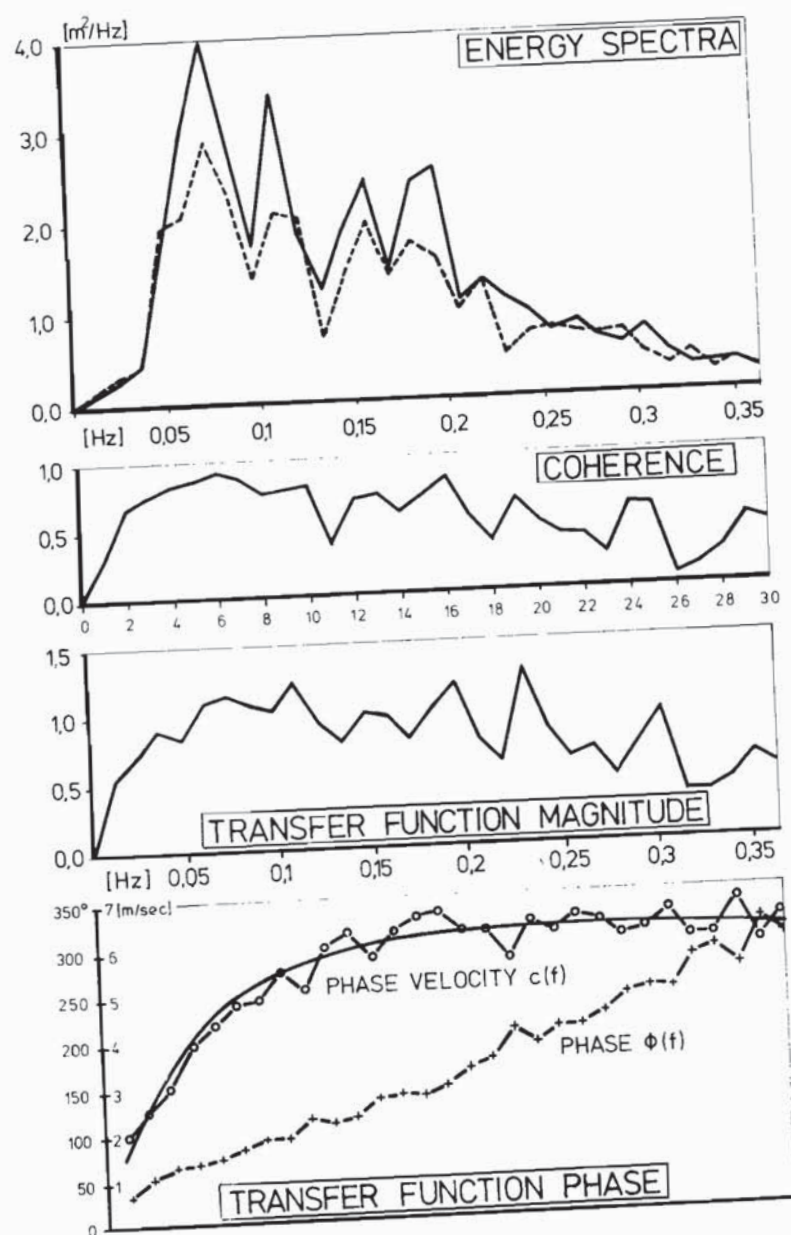


Fig. 4 : Spectral functions from water-level-deflection-measurements on December 12th, 1973, 3.46 a.m.

FOURIER components (near the spectral peak) roughly corresponds to that predicted by the dispersion formula or the shallow water relation. If alternatively phase angles $\phi \pm 360^\circ$ were considered for the transformation quite unrealistic phase velocities would turn out in the present case.

During the storm surge period of about 30 hours in total at 16 measuring intervals samples were analyzed as illustrated in Fig. 4.

The results of those measurements in the form of MEAN VALUES of coherence, transfer function magnitude and phase velocity are plotted in Fig. 5 along with the storm tide curve.

The mean values are related to the frequency ranges as listed in the inset. In case the smallest frequency range only containing the lowest frequency FOURIER components

$$(7) \quad 0 < f \leq 0.06 \text{ Hz}$$

is disregarded, because the confidence of the FOURIER ANALYSIS is not sufficient at low frequencies, the overall impression is that mean values of coherence $\bar{\gamma}^2$ and transfer function magnitude decrease with an increasing number of contributing frequency components (say from curves (6) to curves (2')), whereas the mean values of the phase velocities increase from curve (6) to curve (2').

This means that a linear relationship would rather exist at lower frequency FOURIER components and especially at high water levels, which should be expected.

By contrast on an average the higher frequency components are faster than the longer ones.

The later phenomenon is analyzed more detailed in the following, whereas some remarks regarding the transfer function magnitude are contained in BÜSCHING (1978).

Fig. 6 contains two different sets of curves in the respective frequency band up to 0.36 Hz :

1. A more regular set of curves representing a presentation of the well known DISPERSION FORMULA for the wave propagation

$$c = \frac{L}{T} = L \cdot f = \left(\frac{g \cdot L}{2 \pi} \tanh \frac{2 \pi d}{L} \right)^{1/2}$$

The respective phase velocity curves $c(f)$ are marked by the parameter of the water depth $d[m]$ at the left hand side ; and

2. The set of phase velocity curves $c(f)$, resulting from the storm surge measurements smoothed by non-linear regressions.

As is well known the above dispersion relationship describes a so-called NORMAL DISPERSION : Lower frequency components travel faster than the higher ones ; only in very shallow water all components travel with nearly the same speed. By contrast this does not come out of the storm surge measurements in the near shore area. Here a so-called ANOMALOUS DISPERSION turns out

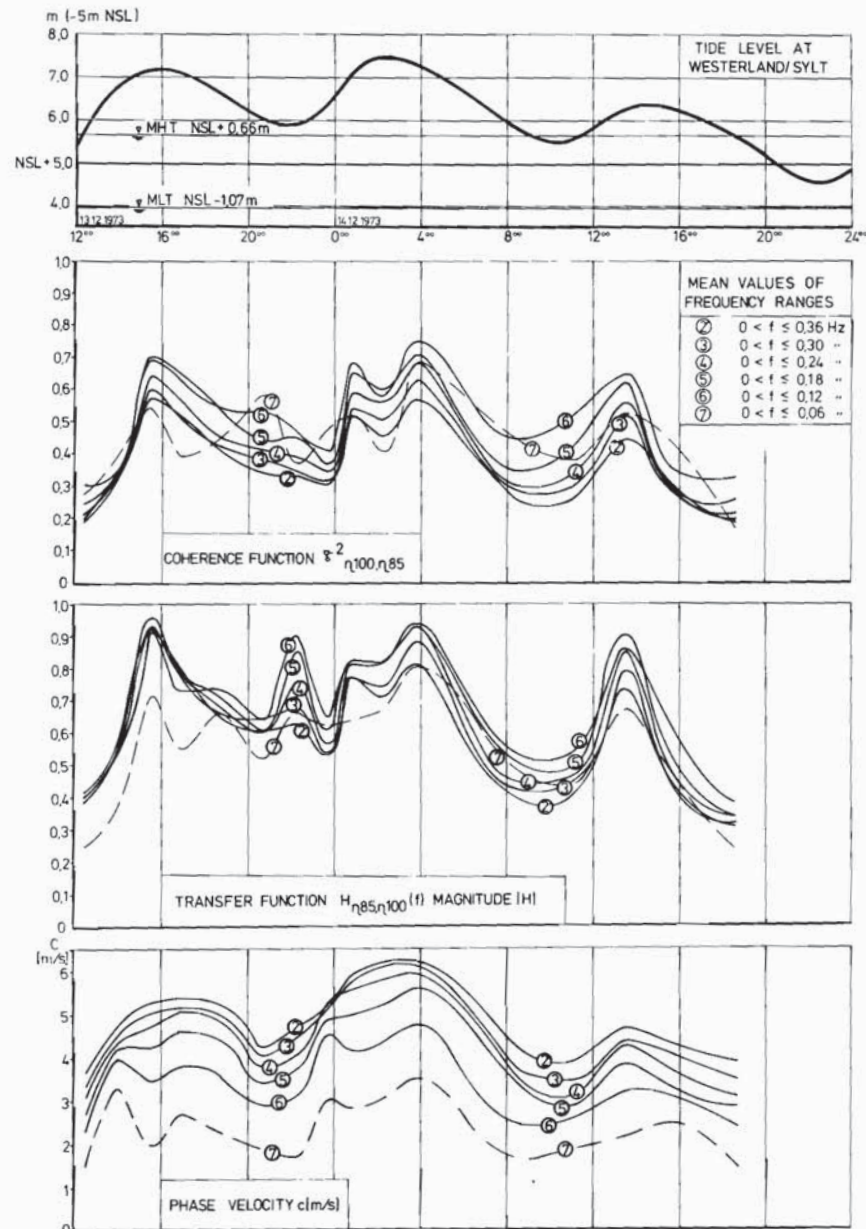


Fig. 5: Mean values of COHERENCE, TRANSFER FUNCTION MAGNITUDE and PHASE VELOCITY of the measurements on December 13th and 14th, 1978

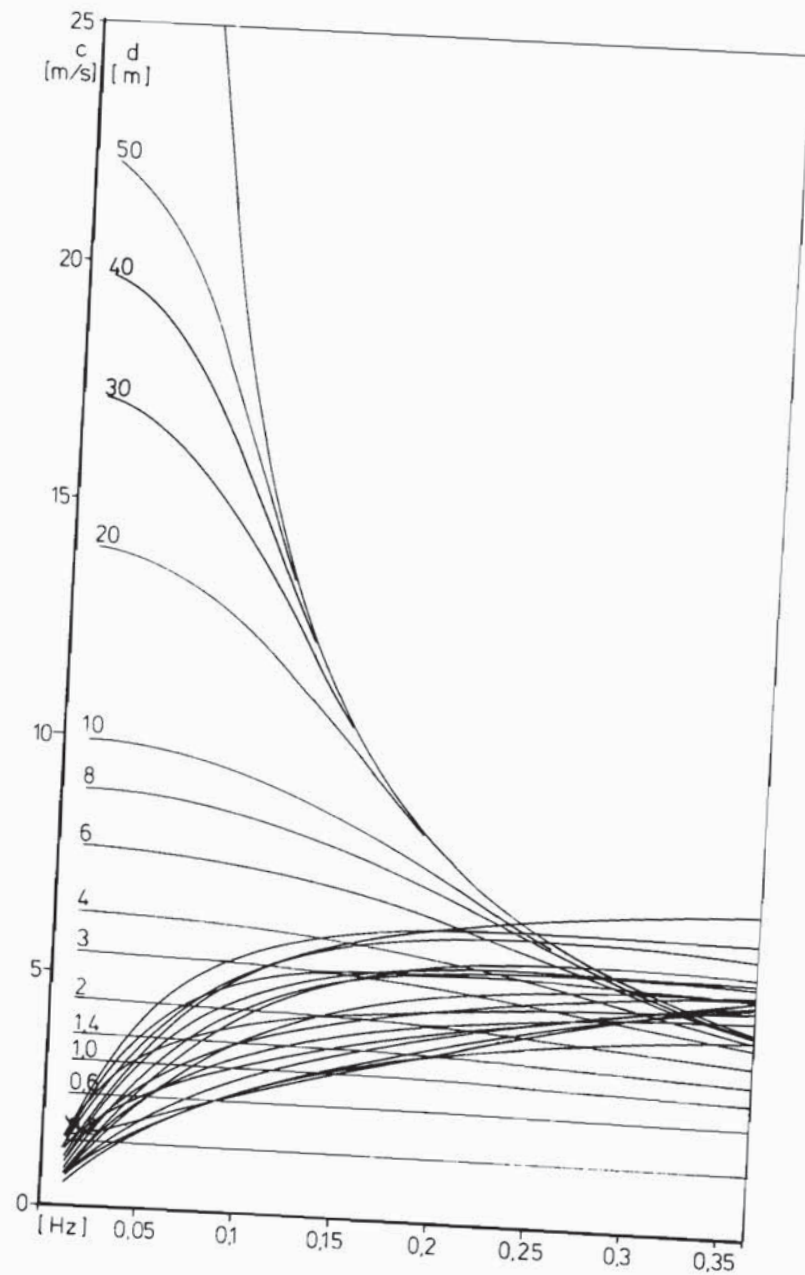


Fig. 6: Predicted and measured dispersion of phase velocities

in the shown frequency band. Even if in the example of Fig. 4 the range of highest coherence, say greater than $\bar{\gamma}^2 \geq 0,8$ in between 0.05 and 0.08 Hz is regarded as reliable only, the statement remains true: In very shallow water the higher frequency components travel faster than the longer ones (in the shown frequency band). In addition it is remarkable that the frequency points of the phase-velocity-plots $c(f)$ form smooth curves wherever the coherence shows appreciable values.

Hence, those components can possibly be regarded as anyhow coupled components of predominating wave systems approaching the beach more or less perpendicularly, whereas especially at very low frequencies (right next to $f = 0$) and at higher frequencies the rate of scatter in the frequency points of the phase information gradually increases with the water depth decreasing. Because of the low coherence in those frequency ranges it is, however, not worthwhile to try an analysis.

According to similar measurements on a platform, standing in a water depth of about $d = 30$ m even at that position about 100 km offshore exists a distinct deviation from the conventional dispersion relationship. As to be seen from Fig. 7 representing a swell measurement on November 22nd, 1975 ($2,0 \leq H \leq 2,5$ m) a distinct normal dispersion exists only on the right hand side of the spectral peak at a frequency $f = 0,12$ Hz, which corresponds to a coherence of $\bar{\gamma}^2 > 0,94$. On the left hand side, however, the tendency for an ANOMALOUS DISPERSION exists, which is at least reliable at high coherence values as indicated in the graph. At lower frequencies again there is a remarkable scatter in the frequency points accompanied by decreasing coherence and phase angles uncertain to determine from the respective plot. In the case of very large phase velocities resulting from very small phase angles at low frequencies it can, however, be supposed that here the phase angle must be changed to $\varnothing + 360^\circ$, see dotted line in Fig. 7.

Comparing this plot to the dispersion formula it turns out that only the phase velocity very near to the spectral peak roughly corresponds to that predicted by the formula. On the right hand side there are higher values to be seen, whereas phase velocities are less on the left hand side.

If high enough frequency components can be regarded as deep water components, which are unaffected by the bottom and the influence of the water depth is regarded to be the dominant parameter for a certain critical frequency f_{crit} at which NORMAL dispersion changes into ANOMALOUS dispersion, it can be stated from this graph that this critical frequency is considerably less than that determined by the condition $d = L/2$ according to linear theory. In the present case

$$f_{crit.} = 0,11 < f_{(d=L/2)} = 0,16 \text{ Hz}$$

With the above findings in mind in the following the behaviour of the FOURIER components in the near shore area are considered again. In order to simplify in the lower part of Fig. 8 the storm surge measurements are represented by 3 smoothed curves only, belonging to water depth $Z_3 = 3,1$ m, $Z_3 = 1,65$ m, $Z_3 = 1,40$ m respectively. As concerns the nearly horizontal portion of the phase velocity curve, corresponding to the maximum water depth $Z_3 = 3,1$ m, this indicates nearly complete non-dispersiveness at frequencies $f \geq 0,2$ Hz.

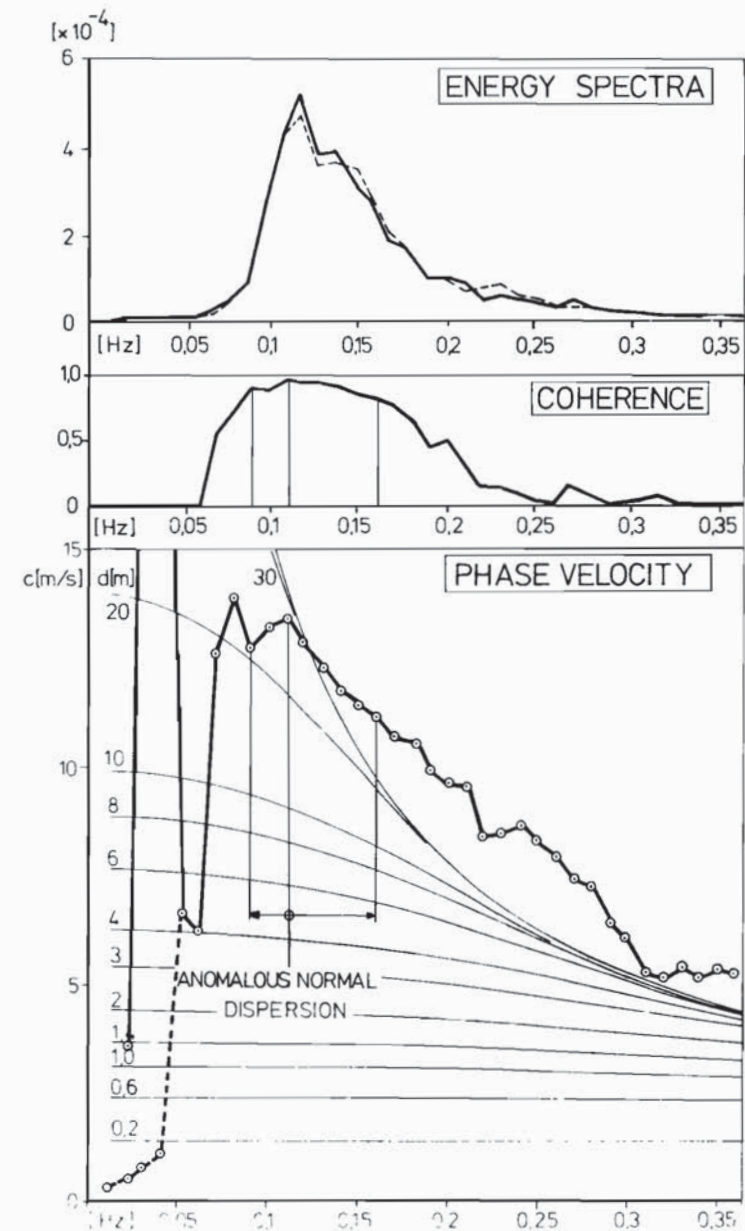


Fig. 7: Spectral functions of a swell-measurement in a water depth $d \approx 30$ m on November 22nd, 1975

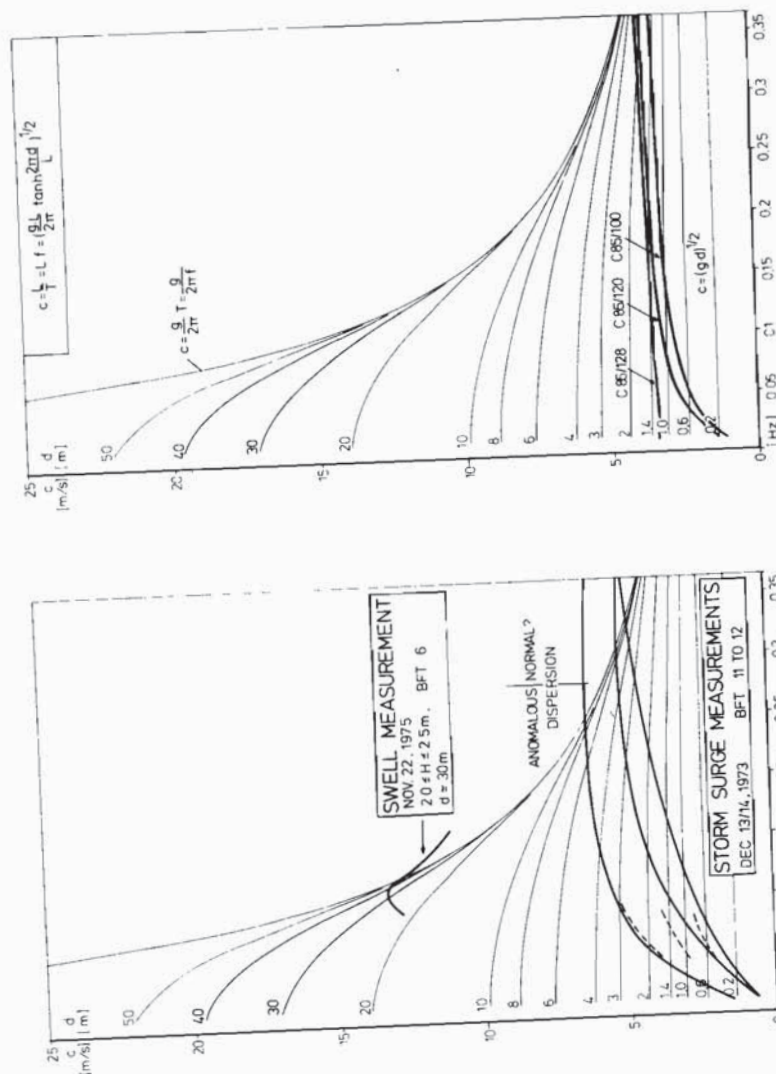


Fig. 8: Normal and anomalous dispersion of phase velocities

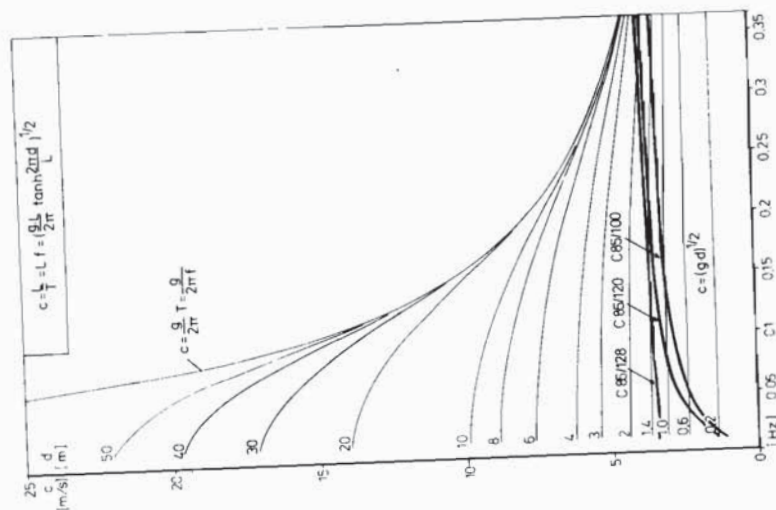


Fig. 9: Weak anomalous dispersion of phase velocities at attenuating wave action, January 20th, 1974

This, however, can also be interpreted as the frequency range where normal dispersion changes to anomalous dispersion. The respective critical frequency f_{crit} is still to be found in the shown frequency band, whereas that of the condition $d = L/2$ according to linear theory is not:

$$0.25 < f_{crit} < 0.30 \text{ Hz} < f_{(d=L/2)} = 0.5 \text{ Hz}$$

In accordance with the even higher critical frequencies ($f_{(d=L/2)} = 0.69 \text{ Hz}$ and $f_{(d=L/2)} = 0.75 \text{ Hz}$ for deep water components at the lower water depth $Z_3 = 1.65 \text{ m}$ and $Z_3 = 1.40 \text{ m}$ respectively, the curves are steeper in the shown frequency band. Hence, the ANOMALOUS DISPERSION in this frequency band gets even more distinct with the water depth decreasing.

This tendency also turns out from the dotted lines representing the respective ranges of maximum coherence values only.

On the other hand it can as well be presumed from this plot a position just seaward of the breakers where the non-dispersive property is even more distinct than at the water depth $Z_3 = 3.1 \text{ m}$. Presumably such a position depends on the actual ratio H/d .

Actually this kind of PHASE COUPLING comes out of the measurements of THORNTON, GALVIN, BUB and RICHARDSON (1976) and this condition is almost included in the profile measurement on January 20th, 1974 carried out at attenuating wave action, see Fig. 9.

The three curves shown result from synchronous measurements at positions 128 m, 120 m, and 100 m with reference to position 85 m. Hence, the curves can be attributed to positions 106.5 m, 102.5 m and 92.5 m respectively.

Compared to the storm surge measurements the dispersion in this case is minimal in the total frequency range at position 106.5 m and with the water depth decreasing the weak dispersive property is preserved at frequencies $f > 0.1 \text{ Hz}$, where in the present case appreciable values of energy density are to be found in the respective energy spectra, see BÜSCHING, 1976.

In order to demonstrate the so-called ANOMALOUS DISPERSION with its effect on the deformation of the waves, Fig. 10 refers to the above mentioned profile measurement of January 20th, 1974 with the weak anomalous dispersive property only.

Assumed that at a certain position outside the shown profile all components are in phase and from that moment on propagate with their own singular phase speeds, one can perform a FOURIER SYNTHESIS after a certain time of propagation, which is 10 sec in the present case. In order to clarify this in Fig. 10 the 0.1 Hz-FOURIER-component is to be seen fixed at the three positions mentioned above.

Additionally only the 0.3 Hz-component is plotted. It is obvious that already at the position 106.5 m the 0.3 Hz-component leads the 0.1 Hz-component and the relative position of the 0.3 Hz-component is shifted in the upbeach direction (with the water depth decreasing). The total FOURIER synthesis consisting of the superimposition of 30 components representing the frequency range

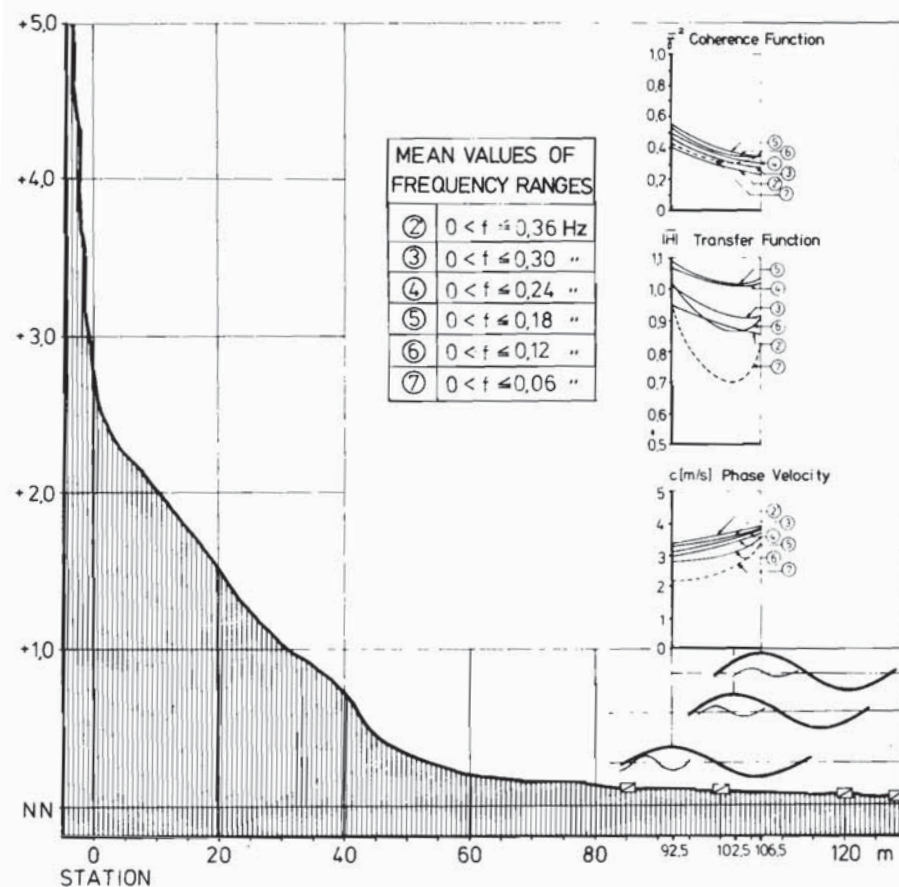


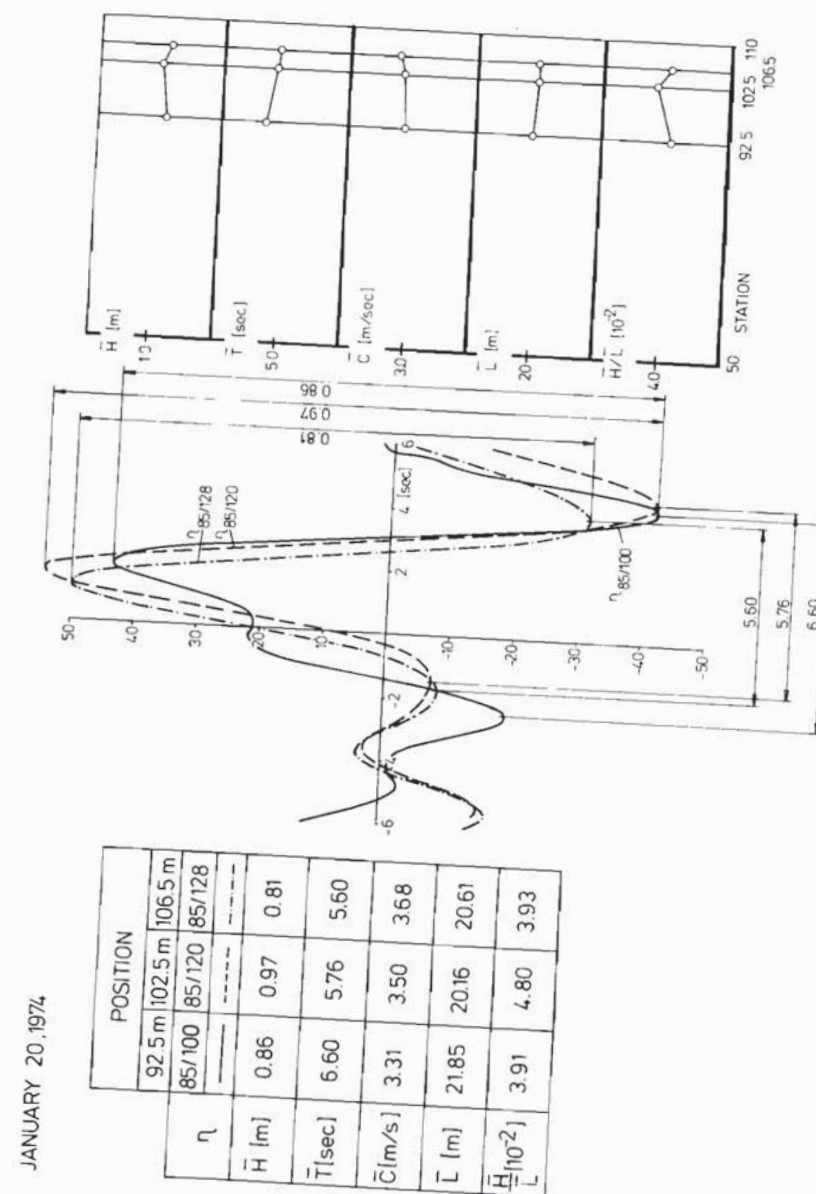
Fig. 10: Relative behaviour of the frequency components 0.1 Hz and 0.3 Hz at decreasing water depth on January 20th, 1974

$0 < f \leq 0.36 \text{ Hz}$ delivers the average wave deformation in the time domain, see middle part of Fig. 11.

The total set of varying wave parameters is to be seen on the right and left hand side of this figure respectively.

Wave heights and periods are taken from the graph as indicated, whereas the variation of the average wave propagation velocity is assumed to be equal to the phase velocity of the peak frequency component.

Hence, the wave length and steepness could also be calculated.



JANUARY 20, 1974

Fig. 11: Average wave deformation resulting from a FOURIER synthesis considering anomalous dispersion of frequency components

4. CONCLUSION

Comparing the results from the time domain wave analysis of the measurements on March 18th and April 3rd, 1973 (Figure 2 and 3) with the results from the FOURIER synthesis considering anomalous dispersion (from the measurements of January 20th, 1974, see Fig. 11) it can be stated that both treatments qualitatively deliver similar variations of the wave parameters.

With the water depth decreasing in the upbeach direction

- a) the wave height \bar{H} at first increases and then continuously decreases,
- b) the wave period \bar{T} increases,
- c) the wave velocity \bar{c} decreases,
- d) the wave length \bar{L} at first decreases and after wave breaking increases and as a result of a) and d)
- e) the steepness \bar{H} / \bar{L} at first increases and then decreases.

The findings according to a), c) and e) are in agreement with the conservative treatment of near shore processes, whereas the increase of the wave period with the consequence of the shown behaviour of the wave length is not. In the following this discrepancy is tried to be explained in terms of a spectral treatment :

Assumed that at idealized conditions there exists a swell in deep water propagating towards the shore (with a normal dispersion), its content of higher order harmonics decreases the more it travels away from its origin.

With the water depth decreasing the swell is increasingly affected by the bottom in such a way that at first only the longer components decrease in phase velocity. As the higher frequency components, being deep water components, keep their phase velocities they recover the more the longer components slow down. At a certain position possibly more or less all of the components travel with approximately the same speed, which means nondispersiveness.

The above described behaviour, which is qualitatively in accordance with the conventional dispersion relationship, at first represents increasingly shortening waves and finally the non-dispersive property leads to high and steep (shoaling) waves, which would not change form very much, if the bottom would be plane at this position.

With the water depth, however, further decreasing the higher frequency components pass the longer ones and thus in a superimposition once again longer waves are produced.

As regards the breaking process the differences between the phase velocities of higher and longer components are such that a regular wave shape can no longer be preserved. The kinetic energy of the continuously slowed down longer components possibly is gradually transferred into potential energy causing a water level increase (wave set up), whereas at breaking that part of kinetic energy associated with the unaffected higher frequencies leaves the former wave shape producing foamlines.

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An explanation like this would be in accordance with the author's previous finding of increasing energy densities at higher frequencies at the expense of the energy densities associated with lower frequencies (BÜSCHING, 1976).

In addition, with respect to Fig. 11 it is obvious that the FOURIER synthesis can better describe the deformation of shoaling waves than at least by means of parameters derived from the zero-up-crossing method as to be seen from Fig. 3.

It clearly turns out that the maximum steepness does not occur together with the maximum asymmetry, which is indicated (in this case) by a continuously steepening front face (Fig. 11). Furthermore finally there exists a saddle on the rear face of the wave, which can be compared very well to the formation of solitons.

In this connection it shall be mentioned here that an anomalous dispersion does not necessarily contradict GALVIN's (1972) wave tank observations stating that in shallow water initially sinusoidal waves decompose into two or more waves (solitons) which travel at different speeds in such a way that the bigger travel faster than the smaller and the crest elevation and the height of the bigger increases.

In this respect a definition is needed between "bigger" and "smaller" waves in terms of a spectral treatment, as it can very well be possible that the longer frequency components are associated with the smaller resultant water level deflexion if the respective energy densities are less at those frequencies.

Hence, being able to reconstruct significant features of the input water level deflexions, this may be an indication that the respective result (ANOMALOUS DISPERSION) obtained by a linear analysis might not be that bad.

Finally some important consequences of an anomalous dispersion associated with energy dissipation, being a very common phenomenon in physics, shall be pointed out :

- 1) As is well known from capillary waves the group velocity of anomalously propagating waves exceeds the phase velocity and thus the energy propagates faster than the singular FOURIER components of a wave system. As concerns gravity waves this may be an indication why the wave energy is transmitted so rapidly, - especially in plunging breakers. Possibly this process can be compared on the one hand with the strong attenuation of capillary waves and on the other hand with the absorption of electromagnetic waves, although the mechanisms - except anomalous dispersion - are completely different.
- 2) In a dispersive system small nonlinear disturbances travel with different velocities and thus an accumulation is more or less prevented. (LIGHT-HILL, 1978). So, at least, the question is open about the rates of contribution on wave deformation by nonlinear and dispersive properties.

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